Mathematics Detailed Performance Level Descriptors

Grades 9 and 10

Performance Level Descriptors
Secondary Mathematics I & II

PLD Type	Below Proficient	Approaching Proficient	Proficient	Highly Proficient
Policy	The Level 1 students are below proficient in applying the mathematics knowledge/skills as specified in the Utah Core State Standards. The students generally perform significantly below the standard for their grade level and are likely able to partially access grade-level content with extensive support.	The Level 2 students are approaching proficient in applying the mathematics knowledge/skills as specified in the Utah Core State Standards. The students generally perform slightly below the standard for their grade level and are likely able to access grade-level content with moderate support.	The Level 3 students are proficient in applying the mathematics knowledge/skills as specified in the Utah Core State Standards. The students generally perform at the standard for their grade level and are able to access grade-level content with minimal support. This level of mathematics performance likely indicates students are on track to be sufficiently prepared for college or career.	The Level 4 students are highly proficient in applying the mathematics knowledge/skills as specified in the Utah Core State Standards. The students generally perform significantly above the standard for their grade level and are able to access above grade-level content. This level of mathematics performance likely indicates students are on track to be well-prepared for college or career.

Secondary Mathematics I

ALGEBRA					
		The Level One Student:	**The Level Two Student:**	**The Level Three Student:**	**The Level Four Student:**
Range	A.SSE.1a, b	Identifies some of the basic terms (base, exponent, coefficient, and factor) of a linear or exponential expression.	Identifies all of the basic terms (base, exponent, coefficient, and factor) of linear and exponential expressions.	Interprets complicated expressions by viewing one or more of their parts as a single entity.	Explains the context of different parts of a formula presented as a complicated expression.
Range	A.CED.1	Creates one variable linear equations and inequalities from contextual situation of a form $2x = 6$ or $3x < 6$.	Creates one-variable linear equations and inequalities from contextual situations of a form $2x + 3 = 7$ or $2x - 5 > 6$.	Creates multi-step linear equations, inequalities, and simple exponential functions in context.	Uses properties of exponents to solve and interpret the solution to multi-step exponential equations and inequalities in context.
Range	A.CED.2	Writes and graphs an equation to represent a linear relationship.	Writes and graphs an equation to represent an exponential relationship.	Constructs equations and graphs that represents linear and exponential relationships. Graphs will include appropriate labels and scales.	Compares and contrasts equations and graphs that model linear and exponential relationships.
Range	A.CED.3	Determines whether a point is a solution to a system of equations and/or inequalities given a graph or equations.	Interprets solutions as viable or non-viable options where constraints are presented in a modeling context.	Represents constraints by equations or inequalities, and by systems of equations and/or inequalities.	Creates a model, defends and justifies solutions or non-solutions in context.
Range	A.CED.4	Rearranges a linear equation that contains only one variable.	Rearranges a linear equation that includes several steps with scaffolding.	Uses linear equation solving techniques to rearrange formulas to highlight a specific quantity by extending concepts used in solving numerical equations.	Identifies useful quantities to highlight the variable of interest and applies the rearranged linear formula to solve problems in context.
Range	A.REI.1	Solves a linear equation without justifying the steps involved in solving.	Describes the steps in solving linear equations.	Explains and justifies the steps in solving linear equations by applying the properties of equality, inverse, and identity.	Explains and justifies the steps in solving linear equations by applying and naming the properties of equality, inverse, and identity.

Range	A.REI.3a, b, c	Solves linear equations and inequalities in one variable.	Solves linear equations and compound inequalities in one variable.	Solves absolute value inequalities and simple exponential equations.	Solves equations and inequalities within a real-world context.
Range	A.REI.5	Explains the use of the multiplication property of equality to solve a system of equations.	Explains why the sum of two equations is justifiable in the solving of a system of equations.	Relates the process of the sum of an equation and a multiple of another equation with the process of substitution for solving a system of linear equations.	Proves that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
Range	A.REI.6	Solves a system of linear equations approximately when given a graph of the system.	Tests a solution to the system in both original equations both graphically and algebraically).	Solves a system of linear equations exactly and approximately by choosing the best method depending on the representation of the equations.	Analyzes the system of equations and is able to solve exactly and approximately given a context or real-world situation. Solves a system of equations and manipulates one of the equations to provide additional information or an additional given solution.
Range	A.REI.10	Given a graph identifies solutions and non-solutions of linear equations in two variables.	Given a graph identifies solutions and non-solutions of exponential equations in two variables.	Graphs points that satisfy linear and exponential equations.	Describes viable solutions using the knowledge that continuous lines and curves contain an infinite number of solutions.
Range	A.REI.11	Finds the point where two lines or exponential curves intersect on a graph or approximates solutions using other methods such as a table or technology.	Finds and explains why the solution to a system of linear/exponential equations is the point where the two functions intersect.	Models the solutions of a system of linear equations and/or exponential equations showing the solutions using technology, tables, graphs, approximations.	Explains why there are infinitely many solutions when $f(x) = g(x)$.
Range	A.REI.12	Identifies a solution region when the graph of a linear inequality is given.	Graphs the solutions to a linear inequality in two variables as a half-plane.	Graphs solutions of the system of inequalities and identifies the solution set as a region of the coordinate plane that satisfies both inequalities.	Writes or creates a system of linear inequalities given a context or graph and identifies the solution set as a region of the coordinate plane that satisfies all inequalities.

FUNCTIONS					
		The Level One Student:	**The Level Two Student:**	**The Level Three Student:**	**The Level Four Student:**
Range	F.IF.1	Identifies functions, including functions represented in equations, tables, or graphs.	Understands domain and range.	Demonstrates understanding that if f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x.	Applies and extends knowledge of domain and range to real world situations and contexts.
Range	F.IF.2	Understands that when an equation is a function it can be written as y equals f(x).	Understands domain and range in relation to function notation.	Uses function notation and evaluates functions for inputs in their domain, and interprets statements that use function notation in terms of context.	Creates context from a given domain and range and uses function notation to write an equation to model the context.
Range	F.IF.3	Identifies arithmetic and geometric sequences.	Identify a recursive sequence as a function.	Writes arithmetic and geometric sequences recursively. Recognizes that a sequence has a domain which is the subset of integers and can generate a sequence given a recursive function.	Applies the ideas of sequences being functions to real world contexts.
Range	F.IF.4	Identifies the key features (as listed in the Standard) when given a linear or exponential graph.	Interprets the key features (as listed in the Standard) when given a linear or exponential graph.	Identifies and interprets the key features (as listed in the Standard) when given a table of values. Sketch graphs of linear or exponential functions, showing key features.	Creates tables or sketches graphs of linear of exponential function, showing key features (as listed in the Standard), when given a verbal description of the relationship.
Range	F.IF.5	Identifies domains of functions when given a graph.	Relates the domain of a function to its graph and graphs a function given a restricted domain.	Relates the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Graphs a function given a restricted domain and identifies reasonability of a domain in a particular context.	Creates a function for a given context where the domain meets given parameters.

Range	F.IF.6	Determines the rate of change of a linear function presented algebraically.	Determines the rate of change of an exponential function presented algebraically over a given interval.	Calculates and interprets the average rate of change of a function (presented symbolically or as a table) over a specified interval.	Interprets the different rates of change over given intervals of a graph.
Range	F.IF.7a	Identifies the graph of a linear function given its equation.	Constructs the graph of a linear function given its equation.	Constructs the graph of a linear function given its equation and identifies the x- and y-intercepts.	Graphs and identifies key features of linear equations generated from real-life contexts.
Range	F.IF.7e	Identifies the graph of an exponential function given its equation.	Constructs the graph of an exponential function given its equation.	Constructs the graph of an exponential function given its equation and identifies the intercepts and end behavior.	Graphs and Identifies key features of exponential equations generated from real-life contexts.
Range	F.IF.9	Compares slopes and y-intercepts of two linear functions where one is presented graphically and the other is presented in slope-intercept form.	Compares growth rates and intercepts of two exponential functions where one is presented graphically and the other is presented in function notation.	Uses tables, graphs, algebra, and verbal descriptions to compare properties of two functions (linear and/or exponential).	Relates multiple representations to compare properties of two or more functions (linear and/or exponential).
Range	F.BF.1a	Writes an explicit and recursive equation for an arithmetic sequence.	Writes an explicit and recursive equation for a geometric sequence.	Describes steps to model a given linear or exponential context with mathematical representations.	Writes an explicit or recursive expression for a linear or exponential function or recursive process for a given context.
Range	F.BF.1b	Combines linear functions using addition and multiplication.	Combines linear and/or exponential functions using addition and multiplication.	Combines linear and/or exponential functions using addition, subtraction, multiplication, and division.	Combines linear and/or exponential functions using addition, subtraction, multiplication, and division for a given context.

Range	F.BF.2	Recognizes if a sequence is arithmetic, geometric, or neither.	Writes arithmetic and/or geometric sequences with an explicit formula.	Writes arithmetic and geometric sequences both recursively and with an explicit formula.	Writes an explicit or recursive expression for a linear or exponential function or recursive process for a given context.
Range	F.BF.3	Relates the vertical translation of a linear function to its y-intercept.	Performs vertical translations on linear or exponential functions.	Describes what will happen to a linear or exponential function when f(x) is replaced by f(x) + k for different values of k.	Finds the value of k given f(x) replaced by f(x) + k on a graph of linear or exponential functions, including all representations.
Range	F.LE.1a, b, c	Recognizes situations in which one quantity changes at a constant rate per unit interval relative to another.	Recognizes relationships in tables and graphs that can be modeled with linear functions (constant rate of change) and with exponential functions (multiplicative rate of change).	Describes the rate of change per unit as constant or the growth factor as a constant percentage.	Justifies that linear functions grow by equal differences over equal intervals and exponential functions grow by equal factors over equal intervals (example: percent change as simple or compound).
Range	F.LE.2	Constructs linear functions representing arithmetic sequences when given a graph.	Constructs exponential functions representing geometric sequences, given a graph.	Constructs sequences, given input-output pairs, including those in a table.	Constructs sequences, given the description of a relationship.
Range	F.LE.3	Recognizes that linear and exponential functions may have points in common when graphed on the same coordinate plane.	Graphs a linear and exponential function on the same coordinate plane and describes how the graphs compare.	Observes that a quantity increasing exponentially eventually exceeds a quantity increasing linearly using graphs and tables.	Describes and compares the changes of behavior between a linear and an exponential function including the approximate point(s) of intersection.
Range	F.LE.5	Identifies the vertical shift and rate of change given a linear function.	Identify the base value and vertical shifts in an exponential function.	Interprets the base value and vertical shifts in an exponential function of the form f(x) = bx + k, where b is an integer and k can equal zero in terms of context.	Interprets the base value and initial value in an exponential function of the form f(x) = ab^x, where b is an integer, and a can be any positive integer including one in terms of context.

GEOMETRY					
		The Level One Student:	**The Level Two Student:**	**The Level Three Student:**	**The Level Four Student:**
Range	G.CO.1	Identifies an angle, circle, perpendicular lines, parallel lines, and line segment using proper notation.	Defines an angle, circle, perpendicular lines, parallel lines, and line segment using examples and non-examples.	Can explain definitions of an angle, circle, perpendicular lines, parallel lines, and line segment based on the notions of point, line, distance along a line, and distance around a circular arc.	Identifies real-life examples of an angle, circle, perpendicular lines, parallel lines, and line segment using precise definitions.
Range	G.CO.2	Describes reflections, rotations, and translations.	Performs rigid transformations.	Compares rigid and non-rigid transformations in the plane and understands them as functions that take points in the plane as inputs and give other points as outputs.	Symbolically represents functions to describe transformations.
Range	G.CO.3	Distinguishes between rotations and reflections given a rectangle, parallelogram, trapezoid, or regular polygon and its transformation.	Identifies lines and points of symmetry given a rectangle, parallelogram, trapezoid, or regular polygon and its reflection or rotation.	Describes the rotations and reflections that a given rectangle, parallelogram, trapezoid, or regular polygon may use to carry it onto itself.	Identifies a rectangle, parallelogram, trapezoid, or regular polygon that satisfies a description of rotational symmetry or lines of symmetry.
Range	G.CO.4	Identifies rotations, reflections, and translations given a figure and its transformation.	Informally describes rotations, reflections, and translations using examples and non-examples.	Develops definitions of rotations, reflections, and translations using the terms angles, circles, perpendicular lines, parallel lines, and line segments.	Justifies statements about rotations, reflections, and translations on the coordinate plane.
Range	G.CO.5	Performs rotations, reflections, and translations on a given figure.	Identifies a sequence of transformations that will carry a given figure onto another.	Performs rotations, reflections, and translations using a variety of methods. Generates the sequence of transformations that will carry a given figure onto another.	Explains how the order of a sequence of transformations is performed may result in different outcomes.

Range	G.CO.6	Explains transformations of a given figure based on geometric descriptions of rigid motion.	Predicts the effect of a transformation of a given figure based on geometric descriptions of rigid motion.	Verifies the congruence of two figures using transformations of rigid motion.	Justifies the congruence of two complex figures using properties of rigid motion.
Range	G.CO.7	Identifies corresponding pairs of angles or corresponding pairs of sides of two congruent triangles.	Identifies corresponding pairs of angles and corresponding pairs of sides of two congruent triangles.	Shows that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent (CPCTC) using the definition of congruence in terms of rigid motions.	Justifies that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent in a context.
Range	G.CO.8	Identifies corresponding parts of two congruent triangles.	Identifies the minimum conditions necessary for triangle congruence (ASA, SAS, SSS).	Demonstrates how the criteria for triangle congruence (ASA, SAS, SSS) follow from the definition of congruence in terms of rigid motions.	Understands and explains why SSA and AAA do not provide enough evidence for triangle congruence.
Range	G.GPE.4	Uses coordinates to find slopes and/or distances algebraically.	Uses coordinates algebraically to determine whether lines are parallel/perpendicular and/or distances to determine whether or not line segments are congruent.	Uses coordinates to prove simple geometric theorems algebraically.	Justifies theorems about geometric figures algebraically using coordinates.
Range	G.GPE.5	Can explain why the slopes of parallel lines are equal and the slopes of perpendicular lines are negative reciprocals or one that is 0 and the other that is undefined.	Creates the equation of a line that passes through a specific point given its slope.	Creates the equation of a line parallel or perpendicular to a given line that passes through a given point.	Creates the equation of a line parallel or perpendicular to a given line that passes through a given point in a context.
Range	G.GPE.7	Calculates the perimeter of a polygon.	Calculates areas of a rectangle and right triangle given their coordinates.	Calculates areas of any triangle given its coordinates.	Calculates perimeters of polygons and areas of triangles and rectangles using their coordinates from a contextual problem.

STATISTICS AND PROBABILITY					
		The Level One Student:	**The Level Two Student:**	**The Level Three Student:**	**The Level Four Student:**
Range	S.ID.1	Identifies dot plots, histograms, and box plots for a given set of data.	Graphs numerical data on a real number line using dot plots, histograms, and box plots.	Describes and gives a simple interpretation of a graphical representation of data on dot plots, histograms, and box plots.	Determines and justifies which type of data plot on a real number line would be most appropriate for a set of data. Identify advantages and disadvantages of different types of data plots.
Range	S.ID.2	Describes the center of a single set of data or graph using mean and median.	Compares the similarities or differences in shape, center, or spread between two graphs.	Explains similarities and differences using specific measures of center and spread, given two sets of data or two graphs.	Plots data based on situations with multiple data sets, and then compares and discusses using measures of center and spread. Justifies which measure(s) are most appropriate for comparison. Identifies advantages and disadvantages of using each measure of center and spread.
Range	S.ID.3	Identifies shape, center, and spread of a data set.	Identifies and states the effects of existing outliers.	Calculates the weighted average of a distribution and interprets it as a measure of center.	Plots and interprets data based on contextual situations involving outliers, and then compares and discusses center and spread and explores the manipulation of additional data points.
Range	S.ID.6a, b, c	Creates a scatter plot of bivariate data and determines if a plotted data set is approximately linear.	Creates a scatter plot of bivariate data and estimates a linear function that fits the data. Uses this function to solve problems in the context of the data.	Writes a linear function that fits the data. Uses this function to solve problems in the context of the data.	Compares the fit of different functions to data and determines which function has the best fit.
Range	S.ID.7	Identifies a linear model of bivariate data.	Graphs data in a scatter plot. Identifies the slope and y-intercept from the linear model.	Uses a line fitted to the data to interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the situation and data.	Uses the function that best fits the data to interpolate and extrapolate trends in the data.

Range	S.ID.8	Uses a table or graph of a set of data to informally describe a correlation.	Computes the correlation coefficient of a set of linearly-related data using technology.	Interprets the correlation coefficient of a linear fit in the context of a situation. Determines whether the correlation shows a weak positive, strong positive, weak negative, strong negative, or no correlation.	Supports or refutes a hypothesized correlation between two sets of data.
Range	S.ID.9	Defines causation and correlation.	Identifies the existence of or non-existence of causation in the context of a correlated problem.	Distinguishes between causation and correlation in the context of a situation with data.	Supports or refutes claims of causation with the understanding that a strong correlation does not imply causation.

Secondary Mathematics II

NUMBER AND QUANTITY					
		The Level One Student:	**The Level Two Student:**	**The Level Three Student:**	**The Level Four Student:**
Range	N.RN.1	Uses proper notation and uses structure for integer exponents only.	Uses proper notation for radicals in terms of rational exponents, but is unable to explain the meaning.	Explains and uses the meaning of rational exponents in terms of properties of integer exponents, and uses proper notation for radicals in terms of rational exponents.	Proves, uses, and explains the properties of rational exponents (which are an extension of the properties of integer exponents), and extends to real world context.
Range	N.RN.2	Converts radical notation to rational exponent notation.	Identifies equivalent forms of expressions involving rational exponents (but is not able to re-write or find the product of multiple radical expressions).	Rewrites expressions involving radicals and rational exponents using the properties of exponents; identifies equivalent forms of expressions involving rational exponents; and converts radical notation to rational exponent notation.	Compares contexts where radical form is preferable to rational exponents, and vice versa.
Range	N.RN.3	Explains why adding and multiplying two rational numbers results in a rational number.	Explains why adding a rational number to an irrational number results in an irrational number.	Explains why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational. Connects to physical situations (e.g., finding the perimeter of a square of area 2).	Generalizes and develops rules about sums and products of rational numbers, the sum of a rational number and an irrational number, and the product of a nonzero rational number and an irrational number.
Range	N.CN.1	Recognizes that the square root of a negative number is not a real number.	Converts simple "perfect" squares to complex number form (bi), such as the square root of -25 is 5i.	Knows that there is a complex number i such that i^2 = -1, and identifies the proper a + bi form (with a and b real).	Generalizes or develops a rule that explains complex numbers and their properties.

Range	N.CN.2	Adds, subtracts, and multiplies using single operations with complex numbers (e.g.: $4i + 5i = 9i$).	Identifies the property (Commutative, Associative, and Distributive) needed to calculate products and sums of complex numbers.	Uses the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	Generalizes or develops rules for complex numbers. For example, explaining what type of expression results, when given $(a + bi)(c + di)$.
Range	N.CN.7	Understands the meaning of a complex number.	Understands the meaning of a complex number and identifies when quadratic equations will have non-real solutions (but is unable to identify the complex solution).	Solves quadratic equations that have complex solutions.	Creates a quadratic function without x-intercepts, and verifies that the solutions are complex.
Range	N.CN.8	Identifies expanded forms of polynomials with complex numbers.	Expresses a quadratic as a product of two complex factors.	Extends polynomial identities to the complex numbers. Limit to quadratics with real coefficients. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.	Generalizes and develops rules for situations involving factored and expanded forms of polynomials, with complex numbers.
Range	N.CN.9	Explains the definition of the Fundamental Theorem of Algebra.	Explains and shows the Fundamental Theorem of Algebra is true for quadratic equations (using equations with only with real roots).	Knows the Fundamental Theorem of Algebra and shows that it is true for quadratic polynomials.	Identifies what values of a, b, and c will provide rational solutions, irrational solutions, and complex solutions, given $y = ax^2 + bx + c$.

ALGEBRA					
		The Level One Student:	**The Level Two Student:**	**The Level Three Student:**	**The Level Four Student:**
Range	A.SSE.1a, b	Identifies some of the basic terms (base, exponent, coefficient, and factor) of an expression.	Identifies the parts of any quadratic and exponential expressions (not in a context).	Identifies and interprets the parts of quadratic and exponential expressions in terms of their context.	Identifies and interprets parts from a variety of different quadratic and exponential expressions by viewing one or more of their parts as a single entity.
Range	A.SSE.2	Identifies different forms for the same expression.	Justifies different forms of the same expression based on mathematical properties.	Recognizes equivalent forms of algebraic expressions, particularly those involving quadratic and exponential functions, and uses the structure of the expression to identify ways to rewrite it.	Rewrites algebraic expressions, including those involving quadratic and exponential functions, to equivalent forms, using the structure of the expression. Makes generalizations by rewriting expressions in context, using their structure.
Range	A.SSE.3a	Identifies the zeroes of a quadratic expression written in factored form.	Factors a quadratic expression without a leading coefficient.	Factors a quadratic expression to reveal the zeros of the function it defines.	Explains conditions for two, one, and no real roots.
Range	A.SSE.3b	Identifies the maximum or minimum of a function, using the graph.	Identifies the maximum or minimum of a function when given in vertex form.	Completes the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	Completes the square in a quadratic expression (where a is not equal to 1).
Range	A.SSE.3c	Knows the properties of exponents.	Applies the properties of exponents.	Uses the properties of exponents to transform expressions for exponential functions.	Rewrites rational exponents as radicals.
Range	A.APR.1	Adds or subtracts polynomials.	Adds, subtracts, and multiplies polynomials using the distributive property and then simplifies.	Adds, subtracts, and multiplies polynomials and understands closure of polynomials.	Adds, subtracts, and multiplies a quadratic expression in contextual situations.

Range	A.CED.1	Identifies a linear, exponential, quadratic equation or inequality that models a given situation.	Creates one-variable linear, exponential, and quadratic equations and inequalities from contextual situations.	Creates equations and inequalities in one variable and uses them to solve problems. Includes equations arising from linear, exponential, and quadratic functions.	Explains the meaning of solutions, and determines when solutions are valid in reference to context.
Range	A.CED.2	Identifies a quadratic graph that represents relationships between quantities.	Graphs a quadratic function with appropriate scales for the variables.	Creates equations in two or more variables to represent relationships between quantities; graphs equations on coordinate axes with labels and scales (linear, exponential, and quadratic).	Given a contextual situation, identifies and creates a linear, exponential, or quadratic equation to model the situation.
Range	A.CED.4	Rearranges a simple quadratic equation (requiring one step).	Rearranges a simple quadratic equation (requiring two steps).	Rearranges formulas to highlight a quantity of interest, using the same reasoning as in solving equations; extend to formulas involving squared variables.	Decides which variable to solve for or isolate, depending upon the given context or problem-solving situation.
Range	A.REI.4a, b	Solves quadratic equations by inspection (e.g., for $x^2 = 49$).	Solves quadratic equations by one method.	Solves quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.	Determines the most efficient method for solving a quadratic equation and justifies the choice selected.
Range	A.REI.7	Identifies by inspection the number of solutions for a system.	Finds approximate solutions of a system of equations from a graph.	Solves a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	Generalizes the number of solutions, given a system consisting of a linear equation and a quadratic equation.

FUNCTIONS					
		The Level One Student:	**The Level Two Student:**	**The Level Three Student:**	**The Level Four Student:**
Range	F.IF.4	Identifies some key features of a graph or table, such as intercepts, relative maximums and minimums, axes of symmetry, and end behavior.	Identifies most key features of a graph or table, such as intercepts, relative maximums and minimums, axes of symmetry, and end behavior.	Interprets all key features of graphs and tables in terms of the quantities, and sketches graphs showing key features given a verbal description of the relationship.	Creates graphs to model a situation.
Range	F.IF.5	Identifies domains of functions, given a graph.	Identifies a domain in a particular context.	Relates the domain of a function to its graph, and, where applicable, to the quantitative relationship it describes.	Relates the domain of a function to its graph, and, where applicable, to the quantitative relationship it describes. Identifies constraints given contextual situations.
Range	F.IF.6	Identifies the rate of change from a table that models a quadratic over a specific interval.	Estimates the rate of change of a quadratic function from a graph.	Calculates and interprets the average rate of change of a quadratic function over a specified interval. Estimates the rate of change from a graph.	Compares rates of change between different types of functions.
Range	F.IF.7a	Evaluates quadratic functions.	Identifies key features of quadratic graphs when the graph is given.	Graphs quadratic functions, showing intercepts, maxima, and minima. Can graph functions expressed symbolically and can show key features of the graph (by hand in simple cases, and using technology for more complicated cases).	Graphs and compares quadratic functions expressed in various forms.

Range	F.IF.7b	Evaluates piecewise and absolute value functions.	Identifies key features of piecewise and absolute value graphs, when the graph is given.	Graphs piecewise-defined functions and absolute value functions, and shows intercepts, maxima, and minima. Graphs functions expressed symbolically and shows key features of the graph (by hand in simple cases, and using technology for more complicated cases).	Graphs and compares piecewise and absolute value functions in various forms.
Range	F.IF.8a	Factors quadratic functions to find zeros, when zeroes are rational numbers.	Identifies zeroes, extreme values, and symmetry of a quadratic function.	Uses the process of factoring and completing the square to show zeroes, extreme values, and symmetry of the graph, and interprets these in terms of context.	Compares different forms of quadratic functions and identifies advantages of each.
Range	F.IF.8b	Evaluates exponential function.	Identifies key features of exponential functions when the graph is given.	Uses the properties of exponents to interpret expressions for exponential functions.	Compares different forms of exponential functions and identifies advantages of each.
Range	F.IF.9	Compares the properties of two functions of the same representation (e.g.: a table to a table, or an equation to an equation).	Compares the properties of two functions of the same type with different representations (such as a quadratic to a quadratic, but using a table and equation).	Compares properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, compares a quadratic equation to an exponential graph.	Creates and compares functions, given a context.
Range	F.BF.1a	Creates a function describing a linear or exponential relationship.	Creates an explicit or recursive expression for a quadratic function.	Creates an explicit expression, a recursive process, or steps for calculation from a context.	Creates an expression, recursive process, or steps to model with mathematical representations (given a quadratic context).
Range	F.BF.1b	Combines linear and exponential functions using arithmetic operations.	Combines standard function types, using addition and multiplication.	Combines standard function types using arithmetic operations.	Combines linear, exponential, and quadratic functions, using arithmetic operations in a context.

Range	F.BF.3	Performs vertical translations on linear and exponential graphs. Describes what will happen to a linear or exponential function when f(x) is replaced by f(x) + k (for different values of k).	Performs translations on linear, exponential, and quadratic graphs. Identifies the value of k, given f(x) replaced by f(x) + k or kf(x).	Identifies the effect on the graph of replacing f(x) with f(x) + k, kf(x), f(kx), and f(x + k), for specific values of k (both positive and negative); finds the value of k, given linear, exponential, and quadratic graphs.	Recognizes which transformations take away the even nature of a quadratic or absolute value function.
Range	F.LE.3	Compares the values of functions over various intervals.	Observes, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity that is increasing linearly.	Observes using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity that is increasing linearly or quadratically.	Predicts, models, and evaluates different situations that compare linear, quadratic, and exponential functions.
Range	F.TF.8	Identifies the trigonometric ratios in a right triangle (e.g., understands what sin and cos represent).	Finds an unknown trigonometric value by using the Pythagorean Identity.	Proves the Pythagorean Identity $\sin^2 x + \cos^2 x = 1$, and uses it to find basic trig values, given one trig value and the quadrant.	Extends the Pythagorean Identity to prove that trig ratios are constant for similar triangles.

GEOMETRY					
		The Level One Student:	**The Level Two Student:**	**The Level Three Student:**	**The Level Four Student:**
Range	G.CO.9	Describes examples of theorems about lines and angles.	Determines the validity of a given proof of a theorem about lines and angles.	Proves theorems about lines and angles. (Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.)	Applies theorems about lines and angles to a real-life context.
Range	G.CO.10	Describes examples of theorems about triangles.	Determines the validity of a given proof of a theorem about triangles.	Proves theorems about triangles. (Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.)	Applies theorems about triangles to a real-life context.
Range	G.CO.11	Defines theorems about parallelograms.	Determines the validity of a given proof of a theorem about parallelograms.	Proves theorems about parallelograms. (Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.)	Applies theorems about parallelograms to a real-life context.

Range	G.SRT.1a, b	Identifies dilations.	Identifies the scale factors of dilations.	Verifies the properties of dilations given by a center and a scale factor, by understanding that a dilation creates parallel lines and line segments in ratios of the scale factor.	Locates the center of dilation and identifies the scale factor, given a pair of similar figures on a coordinate plane.
Range	G.SRT.2	Identifies corresponding parts of two similar figures.	Determines if two given figures are similar.	Explains that two given figures are similar in terms of similarity transformations.	Proves or disproves that two given figures are similar, using transformations and the definitions of similarity.
Range	G.SRT.3	Identifies similarity transformations.	Identifies triangle similarity by the use of the AA criterion.	Establishes the AA criterion for two triangles to be similar by using the properties of similarity transformations.	Proves that two triangles are similar if two angles of one triangle are congruent to two angles of the other triangle, using the properties of similarity transformations.
Range	G.SRT.4	Defines theorems about triangles.	Determines the validity of statements within a given proof of a theorem about triangles.	Proves theorems about triangles. (Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.)	Applies theorems about triangles to a real-life context.
Range	G.SRT.5	Finds measures of sides and angles of congruent and similar triangles.	Solves problems involving triangles, using congruence and similarity criteria.	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	Proves conjectures about congruence or similarity in geometric figures, using congruence and similarity criteria for triangles. Includes problems from context.
Range	G.SRT.6	Understands that, in similar right triangles, corresponding angles are congruent and ratios of corresponding sides are equal.	Identifies that side ratios in right triangles are properties of the angles in the triangle, and makes connections to sine, cosine and tangent.	Understands that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	Determines the similarity of right triangles by comparing the trigonometric ratios of the corresponding sides.

Range	G.SRT.7	Understands that the acute angles of a right triangle are complementary.	Identifies the relationship between the sine and cosine of the acute angles of a right triangle.	Explains and uses the relationship between the sine and cosine of complementary angles.	Solves for missing angles of right triangles using sine and cosine.
Range	G.SRT.8	Solves right triangles using the Pythagorean Theorem.	Applies the Pythagorean Theorem in real-life and mathematical contexts.	Solves right triangles using trigonometric ratios and the Pythagorean Theorem in applied/contextual problems.	Models solutions to situations, using trigonometric ratios and the Pythagorean Theorem, by constructing equations that can be used to solve the problem. Includes problems from context.
Range	G.C.1	Knows the definition of a circle as a set of points equidistant from a given point.	Recognizes that all circles are similar.	Proves that all circles are similar.	Solves applied math problems, using the fact that all circles are similar.
Range	G.C.2	Identifies inscribed angles, radii, and chords in circles.	Recognizes relationships among inscribed angles, radii, and chords in circles.	Describes relationships among inscribed angles, radii, and chords in circles.	Solves problems using relationships among inscribed angles, radii, and chords in circles.
Range	G.C.3	Identifies inscribed and circumscribed circles of a polygon.	Constructs the inscribed and circumscribed circles of a triangle.	Constructs and proves properties of angles for a quadrilateral inscribed in a circle.	Proves the unique relationships between the angles of a triangle or quadrilateral inscribed in a circle.
Range	G.C.4	Identifies a tangent line from a point outside a given circle to the circle.	Sketches an approximate tangent line from a point outside a given circle to the circle.	Constructs a tangent line from a point outside a given circle to the circle.	Constructs a line that is tangent to two given circles.
Range	G.C.5	Defines a sector area of a circle as a proportion of the entire circle.	Develops the definition of radians as a unit of measure by relating to arc length.	Derives, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and defines the radian measure of the angle as the constant of proportionality; derives the formula for the area of a sector.	Proves that the length of the arc intercepted by an angle is proportional to the radius, with the radian measure of the angle being the constant of proportionality.

Range	G.GPE.1	Identifies the center and radius of a circle, given an equation written in $(x - h)^2 + (y - k)^2 = r^2$ form.	Creates the equation for a circle, when given the center and radius.	Derives the equation of a circle of given center and radius using the Pythagorean Theorem; completes the square to find the center and radius of a circle given by an equation.	Determines the equation of a circle, given points of tangency.
Range	G.GPE.4	Solves problems algebraically using geometric theorems.	Proves simple geometric theorems using coordinates, when given a visual representation on the coordinate plane.	Proves simple geometric theorems algebraically using coordinates, such as proving a point lies on a given circle.	Constructs visual representations on the coordinate plane that meet given conditions for coordinates.
Range	G.GPE.6	Finds the point on a line segment that partitions the segment in a given ratio, given a visual representation of the line segment.	Finds the point on a line segment that partitions the segment in a given ratio, given coordinates for the line segment.	Finds the point on a directed line segment between two given points that partitions the segment in a given ratio.	Constructs a line segment that is partitioned in a given ratio.
Range	G.GMD.1	Identifies and uses the formula for the circumference and area of a circle, and the volume of a cylinder, pyramid, and cone.	Informally describes the formulas for the circumference and area of a circle, volume of a cylinder, pyramid, and cone by the use of dissection arguments.	Explains the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Includes understanding of scale factor for the radius, area, and volume. For example, if the radius doubles, then the area quadruples.	Justifies the formulas for the circumference of a circle; area of a circle; and volume of a cylinder, pyramid, and cone.
Range	G.GMD.3	Substitutes given dimensions into the formulas for the volume of cylinders, pyramids, cones, and spheres.	Computes the volume of cylinders, pyramids, cones, and spheres, given a graphic.	Solves problems using the volume formulas for cylinders, pyramids, cones, and spheres.	Finds the volume of cylinders, pyramids, cones, and spheres in a real-life context.

STATISTICS AND PROBABILITY					
		The Level One Student:	**The Level Two Student:**	**The Level Three Student:**	**The Level Four Student:**
Range	S.ID.5	Explains data in a two-way frequency table.	Creates two-way frequency table showing the relationship between two categorical variables.	Finds and interprets joint marginal and conditional relative frequencies. Recognizes possible associations and trends in the data.	Given a context, interprets, identifies, and describes associations and trends using a two-way frequency table.
Range	S.CP.1	Identifies an event as a subset of a set of outcomes (a sample space).	Identifies or shows relationships between sets of events.	Describes events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events.	Using multiple representations, makes sense of outcomes in context. (For example: unions of all subsets would equal the sample space).
Range	S.CP.4	Constructs two-way frequency tables of data.	Approximates conditional probabilities using two-way frequency tables.	Constructs and interprets two-way frequency tables of data when two categories are associated with each object being classified. Uses the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.	Constructs, interprets, and finds missing values of a two-way frequency table.
Range	S.CP.5	Expresses conditional probabilities using probability notation.	Interprets conditional probabilities and independence in context.	Recognizes and explains the concepts of conditional probability and independence, in everyday language and everyday situations.	Using concepts of conditional probability and independence, extrapolates the meaning behind probabilities that were calculated from real-world context.

Range	S.CP.6	Distinguishes between compound and conditional probability scenarios.	Finds the conditional probability of A, given B as the fraction of B's outcomes that also belong to A, using multiple representations (such as two-way tables, Venn diagrams, tree diagrams or probability notation).	Finds the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.	Compares and contrasts conditional probabilities and compound probabilities. (e.g.; from a table, determine the probability of getting the flu, and then compare that to the probability of getting the flu given the individual never washes their hands).